

# A multiobjective model for passenger train services planning: application to Taiwan's high-speed rail line

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Received 16 June 1998; received in revised form 15 March 1999; accepted 24 March 1999

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## Abstract

This paper develops a multiobjective programming model for the optimal allocation of passenger train services on an intercity high-speed rail line without branches. Minimizing the operator's total operating cost and minimizing the passenger's total travel time loss are the two planning objectives of the model. For a given many-to-many travel demand and a specified operating capacity, the model is solved by a fuzzy mathematical programming approach to determine the best-compromise train service plan, including the train stop-schedule plan, service frequency, and fleet size. An empirical study on the to-be-built high-speed rail system in Taiwan is conducted to demonstrate the effectiveness of the model. The case study shows that an optimal set of stop-schedules can always be generated for a given travel demand. To achieve the best planning outcome, the number and type of stop-schedules should be flexibly planned, and not constrained by specific stopping schemes as often set by the planner. © 2000 Elsevier Science Ltd. All rights reserved.

*Keywords:* Passenger train planning; High-speed rail; Stop-schedule; Multiobjective programming

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## 1. Introduction

The operation of passenger train services is based on regular interval, periodic or cyclic train schedules. The planning of train schedules forms the most crucial task in railroad operations planning (Hooghiemstra, 1996). Mathematical programming methods have been applied to the optimization problems arising in the planning process of train schedules (Assad, 1980a; Harker, 1990; Higgins et al., 1996; Bussieck et al., 1997b). Most of the optimization problems in train

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planning and scheduling are handled by single-objective approaches. The single planning objective is usually constructed from the perspective of either the user (such as time, distance, or service level) or the operator (such as cost, revenue, or capacity). However, the nature of the train-schedule planning problem is inherently multiobjective. This is primarily due to the multiplicity of interests embodied by different stakeholders and social concerns. Multiobjective programming techniques have been developed to provide the decision maker with explicit consideration of the relative values of the objectives which are implicitly made in single-objective approaches (Cohon, 1978).

Current and Min (1986) give a systematic review of the transportation planning literature using multiobjective analysis. These studies generally produce better planning alternatives, mainly because relevant factors can be considered as the planning objectives and evaluated in non-commensurable units. Some recent studies have also shown advantages in dealing with the multiobjective nature of the transportation planning problem, especially in transportation networks (Current et al., 1987), air services planning (Flynn and Ratick, 1988), bus operations planning (Tzeng and Shiau, 1988), airline flight planning (Teodorovic and Krčmar-Nozic, 1989), freight train planning (Fu and Wright, 1994), urban school bus planning (Bowerman et al., 1995), transit network design (Israeli and Ceder, 1996), and transportation investment planning (Teng and Tzeng, 1996). In this paper, we formulate the optimal planning problem of passenger train services on an intercity high-speed rail (HSR) line as a multiobjective programming model from both the passenger's and the operator's viewpoints.

Due to the complexity of the rail operations system, a hierarchically structured planning process is usually applied to generate and maintain train schedules (Assad, 1980b; Harker, 1990; Bussieck et al., 1997a). In a rail network system, the fundamental base of the train schedule is a line plan which determines the number of trains serving the line connecting two terminal stations in a fixed time interval (e.g. in one hour) (Bussieck et al., 1997a, b). For a rail line without branches, passenger train scheduling is mainly concerned with the determination of stop-schedules for all train trips planned (Salzborn, 1969; Assad, 1982).

A stop-schedule for a train trip on a rail line specifies a set or subset of stations at which the train stops. For this class of the train scheduling problem, the most common approach is to find the best alternative from all possible stop-schedules with respect to given performance criteria, using dynamic programming (Assad, 1980a). These stop-schedules are constructed based on various zoning schemes (e.g. Salzborn, 1969; Ghoneim and Wirasinghe, 1986), local versus express service (Nemhauser, 1969), specific stopping patterns (e.g. Sone, 1992, 1994; Claessens, 1994) and various numbers of railcars (e.g. Salzborn, 1970). A number of specific stop-schedules such as all-stop, skip-stop and zone-stop schedules (Eisele, 1968) are thus identified and studied. Research results have shown that zone-based stop-schedules have some advantages over all-stop and skip-stop schedules for the travel demand of many-to-one type on a suburban commuter rail line (Ghoneim and Wirasinghe, 1986).

Despite its proven advantages, the zone-based approach may not be applicable to rail lines other than suburban commuter systems. This is because typical zoning schemes are specifically suited for a rail line where the majority of passenger volume originates or terminates at one or a few city center stations (Sone, 1992). In fact, the empirical study conducted by this paper shows that a combination of various stop-schedule types, in which no specific zoning schemes can be formed, is the best stop-schedule plan for serving passengers with many-to-many origin–destination (O–D) on an intercity HSR line.

In subsequent sections, we first present a hierarchical framework for the planning process of passenger train services on an HSR line. We then formulate the major tactical planning decisions as a multiobjective programming model. Finally an empirical study on the to-be-built Taiwan's HSR system is presented to show how the model works and to examine its effects under various planning scenarios. Specific conclusions on train services planning for Taiwan's HSR line are drawn and discussed.

## 2. Planning of HSR passenger train services

HSR systems have been regarded as the intercity passenger transport mode of choice for selected corridors or routes of up to about 800 km. HSR operation requires much more coordinated planning than traditional rail systems in order to provide high-volume, high-frequency passenger services. In the context of train planning, HSR train trips need to be effectively planned by an optimization-based model.

The distinct planning decisions of providing passenger train services on an intercity HSR line can be hierarchically structured as in Fig. 1. The classification of decision levels is based on the framework proposed by Anthony (1965). This paper focuses on the development of an optimal model for train services planning to support major tactical decisions. Tactical decisions are concerned with the effective allocation of available resources to meet the travel demand. The train service plan determines (a) the number of train trips (frequency of service) required, (b) the stations at which each train trip stops (stop-schedule plan) and (c) the minimum number of trains required (size of fleet). A planning horizon is usually specified to reflect varying operating periods of the day such as peak or off-peak periods for meeting different levels of travel demand. The planning horizon is normally divided into fixed time intervals (e.g. on an hourly basis) to facilitate the construction of train schedules (Salzborn, 1969; Bussieck et al., 1997b). The stop-schedule plan and the service frequency are determined for the specified time interval, called operating period. The fleet size is determined for the planning horizon.

The train service plan is drawn based on some strategic decisions, including O–D pairs of travel demand, station settings, operating capacities and planning parameters. The plan is used as a basic guideline to support operational decisions for day-to-day train operations.

Effective planning for providing passenger services requires a balanced view of demand and supply. On the demand side, passenger satisfaction is the key to the successful operation of railroad services. Apart from the safety and comfort factors, convenience is the passenger's major concern when choosing railroad travel. Convenience is related to the service frequency and journey time. In the context of train-schedule planning, train schedules that minimize the passenger's travel time can satisfy this passenger's requirement. The passenger's total travel time on a rail line without branches usually includes waiting time, riding time, and train stopping time. In the tactical planning model for determining optimal train stop-schedules, we need to consider only the train stopping time for reflecting the passenger's travel time loss. This is due to the following settings and assumptions: (a) trains are operated based on fixed and published timetables (in the case of Taiwan's HSR line), and passengers are assumed to arrive at the station accordingly resulting in no waiting time, (b) the waiting time is to be considered by the operational planning

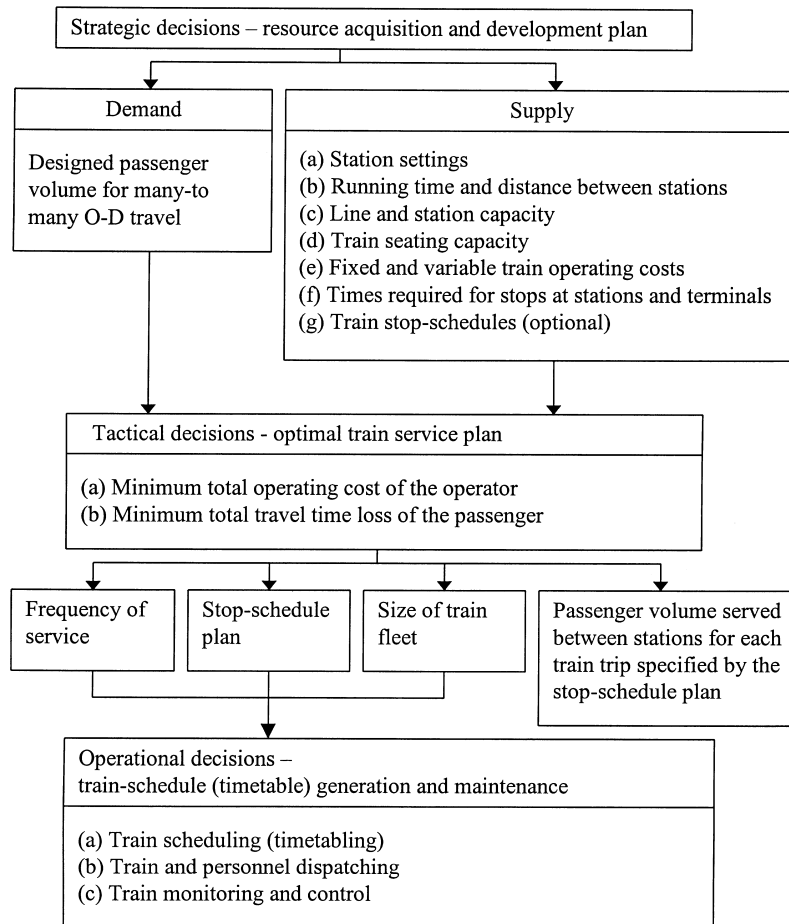


Fig. 1. Planning hierarchy of passenger train services.

model from which detailed timetables are determined, (c) the passenger riding time between any two stations is assumed to be fixed and independent of the stop-schedule and the mode of operation.

On the supply side, the railroad operator would demand the overall operating cost minimized. This can be achieved by a train service plan that optimally allocates minimum train trips required to meet the travel demand. To balance the requirements between the operator and the passenger, two planning objectives are considered: (a) minimizing the operator's total operating cost and (b) minimizing the passenger's total travel time loss. The operating cost of a train trip consists of a fixed overhead cost and a variable cost depending on the trip distance. The operator's total operating cost is defined to be the sum of the fixed and variable operating costs over all train trips that are required to meet the travel demand. There is travel time loss for a passenger if the train stops at an intermediate station at which the passenger does not board or alight. The passenger's total travel time loss is defined as the sum of the time losses for stopping at intermediate stations for all the passengers served by all train trips.

The two planning objectives conflict with each other, and are mainly influenced by the stop-schedules. This objective setting coincides with the two criteria (carriage miles and intermediate passenger stops) used by Salzborn (1969) for determining optimal (zone-based) stop-schedules on a suburban rail line of many-to-one type demand. On an inter-city rail line, the demand pattern of passenger train services is many-to-many. In other words, passengers are picked up at many different origins (stations) and conveyed to many different destinations (stations) within its service line. For meeting a given travel demand during a given operating period, providing only all-stop services will need fewer train trips, thus reducing the total operating cost. However, this will increase the passenger's total travel time due to additional time required for stops at intermediate stations. If express and/or skip-stop services with fewer stops are provided, passengers will spend less travel time. This will increase the total operating cost, as more train trips are required.

Clearly, a combination of various service types realized by a train stop-schedule plan that makes best trade-offs between the minimum operating cost and the minimum travel time loss is required. In the next section, we formulate a multiobjective programming model to generate the best-compromise train service plan for the general passenger train planning problem on an inter-city rail line.

### 3. Model formulation

#### 3.1. The planning problem

On a passenger train service line with a set of  $N$  stations  $\Omega = \{1, 2, \dots, N\}$ , train trips are to be provided by a fleet of  $n$  trains which are operated according to a set of  $R$  stop-schedules within a planning horizon  $T$ . There are a set of shunting stations  $\Phi$  ( $\Phi \in \Omega$ ), including stations 1 and  $N$ , which can be used as a start or end station from which a train trip starts or ends. The running time between any two stations is fixed. The fixed overhead cost  $C_1$  (per train-day) and the variable operating cost  $C_2$  (per train-km) are given, and are independent of the mode of operation. The travel demand of many-to-many O–D for a specified or planned operating period  $t$  ( $t \in \{1, 2, \dots, T\}$ ) (e.g. one hour) over the planning horizon  $T$  is given, and is independent of the mode of operation. The fleet of  $n$  trains is designed to meet the total travel demand  $D_{ijt}$  from station  $i$  ( $i = 1, 2, \dots, N$ ) to station  $j$  ( $j = 1, 2, \dots, N$ ) for the operating period  $t$ .

During an operating period  $t$  ( $t \in \{1, 2, \dots, T\}$ ) with  $H_t$  operating hours, based on a stop-schedule  $r$  ( $r \in \{1, 2, \dots, R\}$ ) a train with seating capacity  $Q_{tr}$  departs from a start station  $s$  ( $s \in \Omega$ ), passes and/or stops at a number of intermediate stations (e.g.  $i, j \in \Omega$ ), and arrives at an end station. The distance and running time between station  $s$  and station  $i$  are  $L_{si}$  and  $U_{si}$  respectively. The time required for stops at station  $i$  is  $W_i$ . During the train trip, there are  $P_{itr}$  passengers on board when the train stops at stations  $i$ , and the passenger volume served between stations  $i$  and  $j$  is  $v_{ijt}$ . Then the train returns to the start station  $s$  following the stop-schedule  $r$  in reverse order, starting from the end station.  $G_r$  terminal time is required for making the round trip. The total trip distance of all train trips based on the stop-schedule  $r$  over the period  $t$  is  $K_{tr}$ .

The planning problem is to determine (a) the optimal train stop-schedule plan, specifying a set of  $R$  stop-schedules, (b) the service frequency  $f_{tr}$  for each stop-schedule  $r$  ( $r = 1, 2, \dots, R$ ), and (c) the minimum operating fleet size  $n$  required for meeting the total travel demand  $D_{ijt}$  within the period  $t$ .

The passenger volume  $v_{ijtr}$  served by each stop-schedule  $r$  is also determined. The planning objectives are to minimize the total operating cost and to minimize the total travel time loss.

To simplify the solution procedure, the problem is formulated as a multiobjective linear programming (MOLP) model. This is achieved by using an arbitrary large constant  $M$  in the model for transforming the nonlinear nature of the two objectives into a linear form. To facilitate this transformation, a stop-schedule  $r$  is represented by a 0-1 integer decision variable  $x_{itr}$ . During the period  $t$ , if a train, based on the stop-schedule  $r$ , stops at station  $i$ , then  $x_{itr} = 1$ ; otherwise,  $x_{itr} = 0$ .

### 3.2. The model

#### Objectives

$$\text{minimize } Z_1 = \sum_{t=1}^T \sum_{r=1}^R C_1 n + C_2 K_{tr} H_t, \quad (1)$$

$$\text{minimize } Z_2 = \sum_{t=1}^T \sum_{r=1}^R \sum_{i=s+1}^{N-1} W_i P_{itr} H_t, \quad (2)$$

$$\text{subject to } K_{tr} \geq 2L_{si} f_{tr} - M(1 - x_{itr}); \quad i = s + 1, \dots, N, \quad (3)$$

$$P_{itr} \geq \sum_{p=s}^{i-1} \sum_{q=i+1}^N v_{pqtr} + \sum_{q=i+1}^N \sum_{p=s}^{i-1} v_{qp tr} - M(1 - x_{itr}); \quad i = s + 1, \dots, N - 1, \quad (4)$$

$$\sum_{r=1}^R v_{ijtr} = D_{ijt}; \quad i = 1, 2, \dots, N; \quad j = 1, 2, \dots, N, \quad (5)$$

$$\sum_{i=s+1}^N x_{itr} \leq (N - s)x_{str}, \quad (6)$$

$$f_{tr} \leq Mx_{str}, \quad (7)$$

$$\sum_{j=1}^N v_{ijtr} + \sum_{j=1}^N v_{jitr} \leq Mx_{itr}; \quad i = s, \dots, N, \quad (8)$$

$$\sum_{r=1}^R f_{tr} \leq E, \quad (9)$$

$$\sum_{p=s}^{j-1} \sum_{q=j}^N v_{pqtr} \leq Q_{tr} f_{tr}; \quad j = s + 1, \dots, N, \quad (10)$$

$$\sum_{p=s}^{j-1} \sum_{q=j}^N v_{qp tr} \leq Q_{tr} f_{tr}; \quad j = s + 1, \dots, N, \quad (11)$$

$$n \geq \sum_{r=1}^R \left( A_{tr} + \sum_{i=s+1}^{N-1} B_{itr} + 2G_r f_{tr} \right), \quad (12)$$

$$A_{tr} \geq 2U_{si}f_{tr} - M(1 - x_{itr}); \quad i = s + 1, \dots, N, \quad (13)$$

$$B_{itr} \geq 2W_i f_{tr} - M(1 - x_{itr}); \quad i = s + 1, \dots, N - 1, \quad (14)$$

where  $t = 1, 2, \dots, T$ ;  $r = 1, 2, \dots, R$ ;  $s \in \Omega$ ;  $M$  is a large constant.

The objective function Eq. (1) is to minimize the total operating cost for the planning horizon  $T$ . This includes (a) total overhead costs of  $n$  trains and (b) total variable operating costs for the trip distance  $K_{tr}$  of each stop-schedule  $r$ , for the operating hours  $H_t$  in each period  $t$ . The objective function (2) is to minimize the passenger's total travel time loss for the planning horizon  $T$ . For each stop-schedule  $r$ , the travel time loss within each period  $t$  is obtained by multiplying the time required for stops ( $W_i$ ) at station  $i$ , the number of passengers on board ( $P_{itr}$ ) at station  $i$ , and the operating hours ( $H_t$ ).

Constraints (3) define the total trip distance  $K_{tr}$  for each stop-schedule  $r$  in the period  $t$  for the objective function (1).  $K_{tr}$  is obtained by multiplying two (for round trips), the distance between station  $s$  and the end station of the stop-schedule  $r$ , and its service frequency  $f_{tr}$  together. The use of the large constant  $M$  and the 0-1 variable  $x_{itr}$  in (3) ensures that the trip distance  $K_{tr}$  obtained is the maximum of the distances  $L_{si}$  between station  $s$  and all stations  $i$  ( $i = s + 1, \dots, N$ ) on the stop-schedule  $r$  (i.e. the distance between station  $s$  and the end station). If a stop-schedule  $r$  is not formed (i.e. all  $x_{itr} = 0$ ), the large constant  $M$  in (3) will make  $K_{tr} = 0$ .

Constraints (4) specify  $P_{itr}$  for the objective function (2).  $P_{itr}$  is the number of passengers on board when the train based on the stop-schedule  $r$  stops at an intermediate station  $i$  ( $i \in \{s + 1, \dots, N - 1\}$ ). It is determined by the passengers who board at a previous station  $p$  ( $p \in \{s, s + 1, \dots, i - 1\}$ ) and alight at a subsequent station  $q$  ( $q \in \{i + 1, \dots, N\}$ ). That is,  $P_{itr}$  is the total passenger volume ( $v_{pgr}$  and  $v_{qpr}$ ) served by the round trip of the stop-schedule  $r$  between stations  $p$  ( $p = s, s + 1, \dots, i - 1$ ) and  $q$  ( $q = i + 1, \dots, N$ ). The large constant  $M$  in (4) makes  $P_{itr} = 0$  when  $x_{itr} = 0$ , for every  $i = s + 1, \dots, N - 1$ . This indicates no travel time loss in (2) as the train does not stop at station  $i$ .

Constraints (5) impose that the travel demand  $D_{ijt}$  between stations  $i$  and  $j$  for the period  $t$  must be met by the total passenger volume  $v_{ijtr}$  served by all stop-schedules  $r$  ( $r = 1, 2, \dots, R$ ). Constraints (6) and (7) specify the conditions for constructing a stop-schedule  $r$ . All train trips based on the stop-schedule  $r$  must start from a start station  $s$  ( $s \in \Omega$ ), where  $x_{str} = 1$ . That is, the train stops at station  $s$ . If  $x_{str} = 0$ , the stop-schedule  $r$  cannot be formed as the train does not stop at station  $s$ . In this case, all 0-1 variables  $x_{itr}$  ( $i = s + 1, \dots, N$ ) in (6) and the service frequency  $f_{tr}$  in (7) are equal to zero. Constraints (8) ensure that no passenger can board or alight at station  $i$  (where  $v_{ijtr} = v_{ijtr} = 0$ ), if a stop-schedule  $r$  does not stop at station  $i$  (where  $x_{itr} = 0$ ).

The capacity constraint for the service line and stations is imposed by (9).  $E$  is the maximum number of trains that can be operated during the period  $t$ . Constraints (10) or (11) state that the total passenger volume served by a one-way trip of the stop-schedule  $r$  ( $v_{pqr}$  or  $v_{qpr}$ ) must be accommodated by the seating capacity of the corresponding train trips ( $Q_{tr} \times f_{tr}$ ).

The minimum operating fleet size  $n$  required is specified by (12).  $n$  is the total number of trains required to provide train trips specified by all stop-schedules. For each stop-schedule  $r$ , the number of trains required is determined by multiplying its total journey time of a round trip by its service frequency  $f_{tr}$ . The total journey time consists of (a) the running time between the start station  $s$  and the end station ( $\max\{U_{si} \times x_{itr}\}_{i=s+1, \dots, N}$ ), (b) the extra time required for stops ( $W_i \times x_{itr}$ ) at all intermediate stations  $i$  ( $i = s + 1, \dots, N - 1$ ) and (c) the terminal time ( $G_r$ ). To trans-

form the two nonlinear items in the calculation (namely, (a)  $f_{tr} \times \max\{U_{si} \times x_{itr}\}_{i=s+1,\dots,N}$  and (b)  $f_{tr} \times W_i \times x_{itr}$ ) into a linear formulation, Constraints (13) and (14) are defined for Constraints (12).  $A_{tr}$  in (13) and  $B_{itr}$  in (14) can be regarded as the number of trains required due to (a) the running time between the start station  $s$  and the end station and (b) the time required for stops at station  $i$  respectively.

### 3.3. *The solution procedure*

Fuzzy mathematical programming has been proven to be an effective approach to an MOLP problem for obtaining the best-compromise solution (Lai and Hwang, 1994). Based on the fuzzy decision of Bellman and Zadeh (1970), the fuzzy feature of this approach lies in the fact that objective functions of the MOLP problem are considered as fuzzy constraints of its equivalent single-objective linear programming (LP) problem. A fuzzy constraint represents the solution space (i.e. feasible solutions) with respect to its corresponding objective function. It is modeled as fuzzy sets whose membership function represents the degree of satisfaction of the objective function. The value of the membership function of an objective function is usually assumed to rise linearly from 0 (for solutions at the least satisfactory value) to 1 (for solutions at the most satisfactory value).

The objective function of the equivalent LP problem is to maximize the overall satisfactory level of compromise between objectives, which is defined by the intersection of membership functions of objective functions of the original MOLP problem. Zimmermann (1978) first uses the max–min operator of Bellman and Zadeh (1970) to aggregate the fuzzy constraints of an LP problem (transformed from all the objective functions of an MOLP problem) for making best-compromise decisions that satisfy both the objectives and constraints of the MOLP problem. The drawback of this operator is that it cannot guarantee a nondominated solution and is not completely compensatory (Lee and Li, 1993). To achieve full compensation between aggregated membership functions of objective functions and to ensure a nondominated solution, we use the augmented max–min approach suggested by Lai and Hwang (1994), which is an extension of Zimmermann's approach. The algorithm for solving the MOLP model for optimal train service planning is given in Appendix A.

With fuzzy mathematical programming, MOLP problems can be solved easily as LP problems. In addition, one advantage of using this approach for solving the optimal train service planning model is that the best-compromise solution will not be affected by the units used for measuring the value of the objectives. As a result of solving the model, the optimal stop-schedule plan, the optimal service frequency, and the optimal fleet size are generated simultaneously. In addition, the passenger volume served between stations by each stop-schedule can be determined.

## 4. Empirical study

### 4.1. *Taiwan's HSR system*

The model developed by this paper for planning passenger train services was motivated by the high-speed rail (HSR) project in Taiwan (Lin, 1995). The proposed HSR system is about a



340-kilometer intercity passenger service line without branches along the western corridor of the island. It connects two major cities, Taipei and Kaohsiung, with 5 or 8 intermediate stations proposed. Strategic decisions regarding the operation plan of the system are made politically and economically by The Bureau of Taiwan High Speed Rail, which are the inputs to the model.

For the 7-station case, Tables 1 and 2 show the relevant data inputted to the model. The planning horizon  $T$  is between 8 am and 6 pm for a normal operating day, and the operating period  $t$  ( $t \in T$ ) is one hour. The terminal time ( $G_r$ ) is required at both ends of a round trip for turning vehicles, making up coaches, and preparing for the return trip or the next round trip. In Taiwan's HSR line, this task is carried out at a depot yard, which is about 15 km away from the terminal station of a trip. The terminal time required thus includes the train running time between the terminal station and the depot yard.

In Table 2, the figures in parenthesis are the distance (km), train running time (min), and planned hourly passenger volume (passenger-trip/h) between two stations respectively. The planned hourly passenger volume is derived from daily forecasted travel demand for the planning horizon of 8 am to 6 pm for 2020.

#### 4.2. Analysis of the optimal train service plan

Table 3 shows the optimal train service plan using data in Tables 1 and 2.

Table 3 illustrates how the best-compromise solution of the MOLP model (minimizing  $Z_1$  and  $Z_2$ ) is obtained by making trade-offs between the optimal solutions for the two single-objectives, respectively. For minimizing the total operating cost, the system would prefer lesser train trips with fewer stop-schedule types. This is because the travel demand is largely met by stop-schedules with more stops. In the case of minimizing the total travel time loss, more train trips for stop-schedules with fewer stops are preferred. In an attempt to make trade-offs between these two conflicting objectives, the best-compromise solution balances the number of train trips and the number of stops in the stop-schedule plan. As a result, it reduces 11.1% of the travel time loss for the optimal solution of minimizing the operating cost, and reduces 3.2% of the operating cost for the optimal solution of minimizing the travel time loss.

Table 1  
Input parameters of the model

Parameter	Value (NT\$: New Taiwan Dollar) (US\$1 $\approx$ NT\$32)
Set of stations ( $\Omega$ )	{1,2,3,4,5,6,7}
Set of shunting or terminal stations ( $\Phi$ )	{1,4,7}
Operating hours ( $H_t$ )	1 h
Terminal time required ( $G_r$ )	45 min/train (round trip)
Train seating capacity ( $Q_{tr}$ )	800 seats/train
Service line and station capacity ( $E$ )	15 trains/h
Extra time required for stops at stations ( $W_i$ )	3 min/station
Fixed overhead cost ( $C_1$ )	NT\$201353/train-day
Variable operating cost ( $C_2$ )	NT\$91459/train-km

Table 2  
Distance, train running time, and planned hourly passenger volume between stations ( $N = 7$ )

Station $i$	1. Taipei	2. Taoyuan	3. Hsinchu	4. Taichung	5. Chiayi	6. Tainan	7. Kaohsiung
1. Taipei	(0,0,0)	(35.8,14,683)	(65.6,27,737)	(159.5,40,1407)	(245.1,65,483)	(307.4,83,636)	(338.1,84,2257)
2. Taoyuan	(35.8,14,697)	(0,0,0)	(29.8,11,149)	(123.7,36,748)	(209.3,61,271)	(271.6,79,305)	(302.3,83,861)
3. Hsinchu	(65.6,27,603)	(29.8,11,111)	(0,0,0)	(93.9,24,320)	(179.5,48,53)	(241.8,67,64)	(272.5,71,242)
4. Taichung	(159.5,40,1298)	(123.7,36,731)	(93.9,24,337)	(0,0,0)	(85.6,22,345)	(147.9,40,413)	(178.6,44,900)
5. Chiayi	(245.1,65,340)	(209.3,61,189)	(179.5,48,45)	(85.6,22,246)	(0,0,0)	(62.3,17,187)	(93.0,29,332)
6. Tainan	(307.4,83,513)	(271.6,79,270)	(241.8,67,68)	(147.9,40,295)	(62.3,17,222)	(0,0,0)	(30.7,11,591)
7. Kaohsiung	(338.1,84,2105)	(302.3,83,776)	(272.5,71,241)	(178.6,44,768)	(93.0,29,465)	(30.7,11,817)	(0,0,0)

Table 3  
Optimal train service plan for the 7-station case

<i>r</i>	Minimizing operating cost ( $Z_1$ )		Minimizing travel time loss ( $Z_2$ )		Minimizing $Z_1$ and $Z_2$	
	Stop-schedule	$f_{tr}$	Stop-schedule	$f_{tr}$	Stop-schedule	$f_{tr}$
1	1 – 7	3	1 – 7	2	1 – 7	2
2	1 – 3 – 4	1	1 – 4	3	1 – 6 – 7	1
3	1 – 4 – 5 – 7	2	1 – 2 – 6 – 7	2	1 – 3 – 4	1
4	1 – 2 – 4 – 6 – 7	3	1 – 2 – 4 – 5 – 7	1	1 – 4 – 5 – 7	2
5	1 – 2 – 3 – 5 – 6 – 7	1	1 – 2 – 3 – 5 – 6 – 7	1	1 – 2 – 3 – 5 – 6 – 7	1
6			1 – 3 – 5 – 6 – 7	2	1 – 2 – 4 – 6 – 7	3
<i>n</i>	42		44		43	
$Z_1$ (NT\$)	14314 613		15000 705		14515 965	
$Z_2$ (h)	9068.00		3708.20		8058.04	

*r*: stop-schedule type;  $f_{tr}$ : service frequency; *n*: fleet size.

The total passenger volume served between stations ( $v_{ijtr}$ ) by each train stop-schedule *r* ( $r = 1, 2, \dots, 6$ ) of the optimal plan indicates that the service type of stop-schedules basically matches the demand pattern of passengers. That is, long-distance passengers are served mostly by express services (e.g.  $r = 1, 2$ ), and short-distance passengers are served mainly by skip-stop (e.g.  $r = 3, 4$ ) or all-stop services (e.g.  $r = 5, 6$ ). This reflects the fact that the model aims to minimize travel time losses by allocating passengers to stop-schedules that stop fewer intermediate stations.

The empirical study has also been carried out for the 10-station case. In terms of effectiveness of the model and the implication of the optimal train service plan, the result is similar to the 7-station case. Stop-schedules with fewer stops should be used to serve stations between which larger passenger volume exists. On the other hand, it is better to use stop-schedules with more stops to serve stations between which there is smaller passenger volume. It is noteworthy that the optimal number of stop-schedule types required by the 10-station case is 6, the same as the 7-station case. This is because the travel demand for the additional 3 stations is relatively small and is quite evenly distributed. This reflects the fact that the stop-schedule plan is largely affected by the volume and pattern of travel demand.

#### 4.3. The optimal train service plan under a given set of stop-schedules

The above study is conducted for situations where the optimal set of stop-schedules is to be determined. In fact, the model can be used to explore various specific train stopping schemes that are often proposed by the planner. That is to say that the model can draw an optimal train service plan from a fixed set of stop-schedules. This is also the planning scenario commonly studied in the literature.

To examine how the stop-schedule plan affects the train service plan in terms of planning objectives, we first limit the number of stop-schedule types in the model for the 7-station case. This has practical implications, as the operator may prefer a stop-schedule plan that is more manageable. This is carried out by solving the model with a specified number of stop-schedule types ( $R$ ). We have examined the value of  $R$  ranging from 1 to 10. For  $R \geq 7$ , the result is the same as

the optimal one in Table 3 (where  $R=6$ ). In other words, only up to 6 stop-schedule types (the optimal number) can be generated, even if  $R$  is specified to be greater than 6. For  $R \leq 6$ , the greater the value of  $R$  specified, the better the plan constructed (in terms of planning objectives and fleet size). This indicates that various stop-schedule types are required to match different travel demand patterns.

We further examine scenarios where the plan is to be drawn from a fixed set of stop-schedules. In the planning of Taiwan's HSR system, 7 stop-schedule types, listed in the second column of Table 4, are considered. Proposed alternative stopping schemes are drawn from combinations of 4–7 stop-schedule types. We first consider the scheme that all 7 types are used. With this given set of 7 stop-schedule types as an input to the model, the optimal plan is shown in the fifth column of Table 4, which is a trade-off between the plans shown in the third and fourth columns.

The service frequency for each stop-schedule of the optimal plan (in column 5 of Table 4) seems to indicate that express or skip-stop services (e.g.  $r=1$  or 6) are preferred to local services (e.g.  $r=5$ ). This is mainly due to the fact that the travel demand of long-distance passengers is greater than that of short-distance passengers.

Although an optimal plan can be drawn from a given set of stop-schedules as often proposed by the planner, it is not normally the best possible planning outcome. This is because the fixed set of stop-schedules may not effectively allocate uneven passenger volumes between stations. For example, the plan in Table 3 compares favorably with that in Table 4 in terms of planning objectives and fleet size. The optimal plan with flexible stop-schedules in Table 3 would reduce 3.5% of the operating cost and 5.8% of the travel time loss, as compared to the optimal plan in Table 4 with all 7 stop-schedule types proposed for Taiwan's HSR line. It is noteworthy that in both cases the optimal number of stop-schedule types ( $R$ ) is 6, although 7 types are specified for the case of given stop-schedules.

To explore how other specific stop-schedule schemes may affect the effectiveness of the optimal plan, we have examined all feasible subsets of stop-schedules listed in the second column of

Table 4  
Optimal train service plan under a given set of stop-schedules

Type ( $r$ )	Stop-schedules (given) Stations stopped	Service frequency ( $f_{tr}$ )		
		Minimizing $Z_1$ (operating cost)	Minimizing $Z_2$ (travel time loss)	Minimizing $Z_1$ and $Z_2$
1	1 – 7	1	3	3
2	1 – 4 – 7	3	2	1
3	1 – 2 – 3 – 4	1	1	0
4	1 – 4 – 5 – 6 – 7	0	1	1
5	1 – 2 – 3 – 4 – 5 – 6 – 7	1	1	1
6	1 – 2 – 4 – 6 – 7	3	2	3
7	1 – 3 – 5 – 7	1	1	1
Train fleet size ( $n$ )		42	46	44
Operating cost ( $Z_1$ ) (NT\$)		14314 610	15738 470	15044 010
Travel time loss ( $Z_2$ ) (h)		12 730.73	8197.37	8554.70

Table 4. The results show that the more the number of stop-schedules in the specified set (regardless of its combination), the better the plan. In comparison with the optimal plan in Table 3, the operating cost increases by 4.8–24.2% and the travel time loss increases by 12.5–96.7% as the given number of stop-schedule types decreases from 6 to 4 for the proposed alternative stopping schemes for Taiwan's HSR line. For cases where the specified set has the same number of stop-schedules, plans which include express services ( $r = 1$ ) perform better. This is because relatively large passenger volume exists between stations 1 and 7, and it is better served by stop-schedules with no intermediate stops.

The findings from the study conducted above suggest that the number and service type of train stop-schedules for Taiwan's HSR line should be flexibly planned based on the volume and pattern of travel demand. The model developed can be used to determine an optimal set of stop-schedules for a given travel demand. If a fixed set of stop-schedules has to be specified for practical reasons, it should be set as close to the optimal set of stop-schedules as possible in terms of the number and/or the service type. In any event, express services between stations 1 and 7 (two major cities) should be provided to meet the relatively large travel demand between the two stations.

## 5. Conclusion

Effective planning of passenger train services requires considering the needs of both the operator and the passenger. In this paper, we have presented a multiobjective programming model for minimizing both the total operating cost of the operator and the total travel time loss of the passenger. The model can be solved using fuzzy mathematical programming to generate a best-compromise train service plan, including the optimal stop-schedule plan, service frequency and fleet size. The empirical study conducted on Taiwan's HSR line has demonstrated the effectiveness of the model. For a given travel demand, the model can be used to determine an optimal set of stop-schedules with or without service types specified. The results of the empirical study suggest that the train service plan of 6 stop-schedule types is the best for Taiwan's HSR line. The best planning outcome is the optimal plan not constrained by specific stop-schedules. In practical planning settings where specific stop-schedules are proposed, the closer the proposed stop-schedules to the optimal set in terms of the number and/or the service type, the better the plan. In particular, express services between the two major cities should be provided for Taiwan's HSR line.

As a planning decision aid, the model can be used to examine various planning scenarios for practical planning purposes. It has general application in planning passenger train services of many-to-many demand on an intercity rail line.

## Acknowledgements

This research was supported in part by the National Science Council of Taiwan under Contract NSC87-2811-E006-0019. We are grateful to anonymous referees for their valuable comments and suggestions.

**Appendix A. Solution procedure for solving the MOLP model**

*Step 1:* Construct the payoff table of the positive-ideal solution by solving the single-objective LP problem with the objective function (1) or (2) (i.e.  $Z_1$  or  $Z_2$ ) individually, as shown in Table 5. The positive-ideal solution is the one that optimizes each objective function simultaneously. In Table 5,  $x$  is the best compromise solution to be found. For the objective function  $Z_1$ ,  $x_1^*$  is the feasible and optimal solution, and  $U_1$  and  $L_1$  are the upper and lower bounds of the solution set.  $x_2^*$  is the feasible and optimal solution, and  $U_2$  and  $L_2$  are the upper and lower bounds of the solution set for the objective function  $Z_2$ .

*Step 2:* Construct the membership functions  $\mu_1(x)$  and  $\mu_2(x)$  for the two objective functions  $Z_1$  and  $Z_2$ , respectively by

$$\mu_1(x) = \begin{cases} 1 & \text{if } Z_1 \leq L_1, \\ 1 - \frac{Z_1 - L_1}{U_1 - L_1} & \text{if } L_1 < Z_1 < U_1, \\ 0 & \text{if } Z_1 \geq U_1, \end{cases} \tag{15}$$

$$\mu_2(x) = \begin{cases} 1 & \text{if } Z_2 \leq L_2, \\ 1 - \frac{Z_2 - L_2}{U_2 - L_2} & \text{if } L_2 < Z_2 < U_2, \\ 0 & \text{if } Z_2 \geq U_2. \end{cases} \tag{16}$$

*Step 3:* Obtain the single-objective LP model by aggregating  $\mu_1(x)$  and  $\mu_2(x)$  using the augmented max–min operator as

Objective

$$\text{maximize } \alpha + \frac{\varepsilon(\mu_1(x) + \mu_2(x))}{2}. \tag{17}$$

$$\text{subject to } \alpha \leq \mu_1(x), \tag{18}$$

$$\alpha \leq \mu_2(x), \tag{19}$$

Objective functions (1)–(2),

Constraints (3)–(16),

where  $\alpha$  is the overall satisfactory level of compromise (to be maximized) and  $\varepsilon$  is a small positive number. A nondominated solution is always generated when  $\alpha$  is maximized. This is because the averaging operator used in the objective function (17) for  $\mu_1(x)$  and  $\mu_2(x)$  is completely compensatory.

The single-objective LP model can be solved using LP software such as LINDO.

Table 5  
Payoff table of positive-ideal solution

	$Z_1(x)$	$Z_2(x)$	$x$
Min $Z_1$	$Z_1(x_1^*)$	$Z_2(x_1^*)$	$x_1^*$
Min $Z_2$	$Z_1(x_2^*)$	$Z_2(x_2^*)$	$x_2^*$
	$U_1 = \max Z_1(x_1^*), Z_1(x_2^*)$	$U_2 = \max Z_2(x_1^*), Z_2(x_2^*)$	
	$L_1 = \min Z_1(x_1^*), Z_1(x_2^*)$	$L_2 = \min Z_2(x_1^*), Z_2(x_2^*)$	

## References

- Anthony, R.N., 1965. *Planning and Control Systems: a Framework for Analysis*. Harvard University, Boston.
- Assad, A.A., 1980a. Models for rail transportation. *Transportation Research* 14 A, 205–220.
- Assad, A.A., 1980b. Modelling of rail networks: toward a route/makeup model. *Transportation Research* 14 B, 101–114.
- Assad, A.A., 1982. A class of train-scheduling problems. *Transportation Science* 16, 281–310.
- Bellman, R.E., Zadeh, L.A., 1970. Decision-making in a fuzzy environment. *Management Science* 17, 141–164.
- Bowerman, R., Hall, B., Calamai, P., 1995. A multi-objective optimization approach to urban school bus routing: Formulation and solution method. *Transportation Research* 29A, 107–123.
- Bussieck, M.R., Kreuzer, P., Zimmermann, U.T., 1997a. Optimal lines for railway systems. *European Journal of Operational Research* 96, 54–63.
- Bussieck, M.R., Winter, T., Zimmermann, U.T., 1997b. Discrete optimization in public rail transport. *Mathematical Programming* 79, 415–444.
- Claessens, M.T., 1994. A mathematical programming model to determine a set of operation lines at minimal costs. In: Murthy, T.K.S., Mellitt, B., Brebbia, C.A., Sciotto, G., Sone, S. (Eds.), *Computers in Railways IV, Railway Operations*, vol. 2. Computational Mechanics Publications, Southampton, UK, pp. 117–123.
- Cohon, J.L., 1978. *Multiobjective Programming and Planning*. Academic Press, New York.
- Current, J., Min, H., 1986. Multiobjective design of transportation networks: Taxonomy and annotation. *European Journal of Operational Research* 26, 187–201.
- Current, J., Revelle, C.S., Cohon, J.L., 1987. The median shortest path problem: a multi-objective approach to analyze cost vs accessibility in the design of transportation networks. *Transportation Science* 21, 188–197.
- Eisele, D.O., 1968. Application of zone theory to a suburban rail transit network. *Traffic Quarterly* 22, 49–67.
- Flynn, J., Ratick, S., 1988. A multiobjective hierarchical covering model for the essential air services program. *Transportation Science* 22, 139–147.
- Fu, Z., Wright, M., 1994. Train plan model for British rail freight services through the channel tunnel. *Journal of the Operational Research Society* 45, 384–391.
- Ghoneim, N.S.A., Wirasinghe, S.C., 1986. Optimum zone structure during peak periods for existing urban rail lines. *Transportation Research* 20B, 7–18.
- Harker, P.T., 1990. The use of ATCS in scheduling and operating railroads: Models, algorithms and applications. *Transportation Research Record* 1263, 101–110.
- Higgins, A., Kozan, E., Ferreira, L., 1996. Optimal scheduling of trains on a single line track. *Transportation Research* 30B, 147–161.
- Hooghiemstra, J.S., 1996. Design of regular interval timetables for strategic and tactical railway planning. In: Allan, J., Brebbia, C.A., Hill, R.J., Sciotto, G., Sone, S. (Eds.), *Computers in Railways V*, vol. 1. Computational Mechanics Publications, Southampton, UK, pp. 393–402.
- Israeli, Y., Ceder, A., 1996. Multi-objective approach for designing transit routes with frequencies. In: Bianco, L., Toth, P. (Eds.), *Advanced Methods in Transportation Analysis*, Springer, Berlin, pp. 157–182.
- Lai, Y-L., Hwang, C-L., 1994. *Fuzzy Multiple Objective Decision Making: Methods and Applications*. Springer, Berlin.
- Lee, E.S., Li, R.J., 1993. Fuzzy multiple objective programming and compromise programming with Pareto optimum. *Fuzzy Sets and Systems* 53, 275–288.
- Lin, C-Y., 1995. The high-speed rail project and its privatization in Taiwan. *Rail International*, pp. 175–179.
- Nemhauser, G.L., 1969. Scheduling local and express trains. *Transportation Science* 3, 164–175.
- Salzborn, E.J.M., 1969. Timetables for a suburban rail transit system. *Transportation Science* 3, 279–316.
- Salzborn, E.J.M., 1970. The minimum fleetsize for a suburban railway system. *Transportation Science* 4, 383–402.
- Sone, S., 1992. Novel train stopping patterns for high-frequency, high-speed train scheduling. In: Murthy, T.K.S., Allan, J., Hill, R.J., Sciotto, G., Sone, S. (Eds.), *Computers in Railways III, Technology*, vol. 2. Computational Mechanics Publications, Southampton, UK, pp. 107–118.
- Sone, S., 1994. High-speed, high-density train allocation. In: Murthy, T.K.S., Mellitt, B., Brebbia, C.A., Sciotto, G., Sone, S. (Eds.), *Computers in Railways IV, Railway Operations*, vol. 2. Computational Mechanics Publications, Southampton, UK, pp. 11–18.

- Teng, J-Y., Tzeng, G-H., 1996. A multiobjective programming approach for selecting non-independent transportation investment alternatives. *Transportation Research* 30B, 291–307.
- Teodorovic, D., Krmar-Nozic, E., 1989. Multicriteria model to determine flight frequencies on an airline network under competitive conditions. *Transportation Science* 23, 14–25.
- Tzeng, G-H., Shiau, T-A., 1988. Multiple objective programming for bus operation: a case study for Taipei city. *Transportation Research* 22B, 195–206.
- Zimmermann, H.J., 1978. Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems* 1, 45–55.